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Module 5

Topic: Triangle-based Explorations

The purpose of this professional development session is to provide teachers the opportunity to explore a typical calculus problem using geometric approaches in an effort to expand their current problem-solving task inventories to include higher level mathematics problems. Teachers will also practice creating and testing conjectures, and consider students work samples to the same problems.

Mathematical Elements

- Consider the relationships between and among triangles and polygons, and associated properties.
- Investigating different approaches (i.e. geometric) to solve probability problems.
- Probability can be interpreted geometrically as proportional to the area defined by constraints.
- Practice creating and testing conjectures
- Triangle area, congruence, similarity, proportion, random selection

Pedagogical Elements

- Classification of problems (i.e. the spaghetti problem is a calculus problem) is not a productive way of thinking about instruction and limits the problems we offer to students.
- Problem-solving tasks can become accessible to all children if different approaches and techniques are used.
- Technology affords the space for conjecturing and testing so students can draw their own inferences.

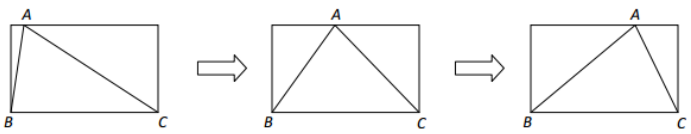
Activities

Task

- A stick of uncooked spaghetti is broken into three pieces by picking two points independently and uniformly along the stick, and breaking the stick at those two points. What is the probability that the three pieces can be assembled to form a triangle?
- Post solution strategies on chart paper.

Task

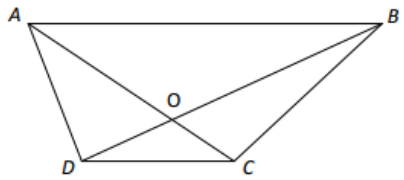
- The area of the rectangle is 1 square inch. Describe how the area of triangle *ABC* would change when *A* moves along the side of the rectangle.



- Discuss student work samples.

Task

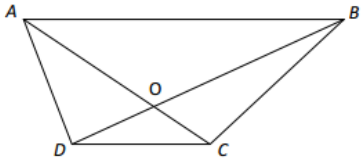
- *ABCD* is a trapezoid with *AB* parallel to *DC*. The diagonals *AC* and *BD* intersect at *O*. Make a few conjectures about the four triangles formed inside trapezoid *ABCD*. Explain how you arrived at these conjectures.



- Discuss student work samples.

Task

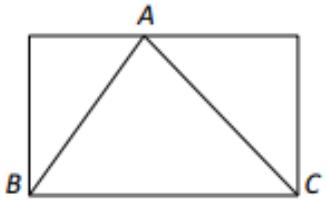
- Dinglei claims that ΔCOB and ΔADO have the same area and that ΔDOC and ΔAOB are similar. However, she does not provide an explanation for her claims. Offer an explanation for why you believe she is right or wrong in each case.



- Discuss student work samples.

Task

- The area of the rectangle is 1 square inch. Assuming ΔABC shares the side *BC* with the rectangle, and vertex *A* of the triangle remains on the opposite side of *BC*, is it possible for the area of ΔABC to be larger than one half of the area of the rectangle?



- Discuss student work samples.

Guided Questions for Debrief

- What other higher-level mathematics problems can be solved using different approaches that we would add to our inventory?
- What questions could teachers ask students that extend the concept horizontally?
- What questions could teachers ask students that extend the concept vertically?

- Which Mathematical Practices were demonstrated with these tasks?

Materials

Technology

- Laptop
- Document Projector
- Document Camera

Supplies

- Chart Paper/Markers
- Copies of handouts for each separate task
- Copies of Student Work Samples handout
- Raw spaghetti noodles

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Triangle-based Explorations

Spaghetti Problem

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Triangle-based Explorations

Changing Area

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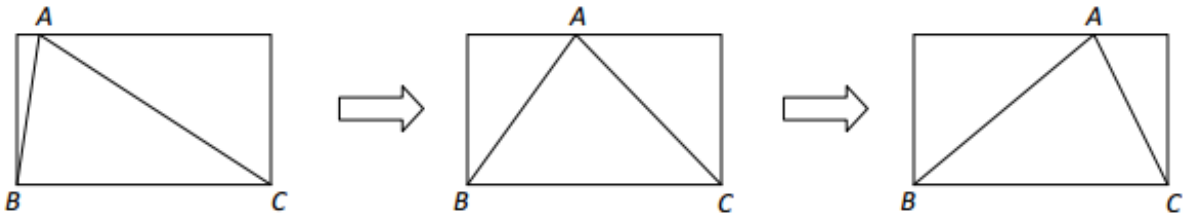

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Student Work Samples

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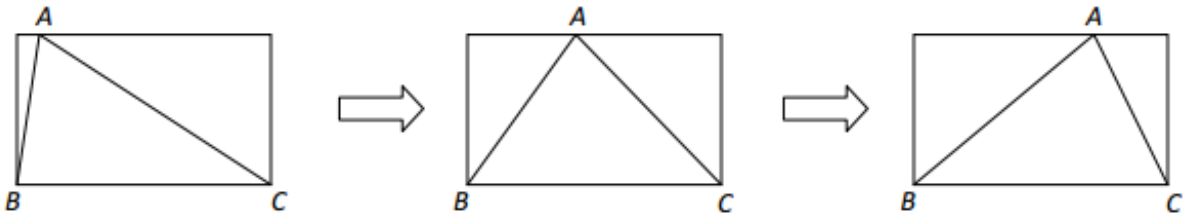

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Triangle-based Explorations

Conjectures about Triangles

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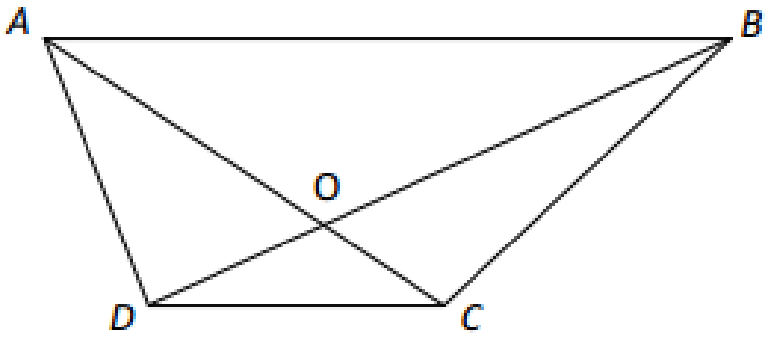

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Student Work Samples

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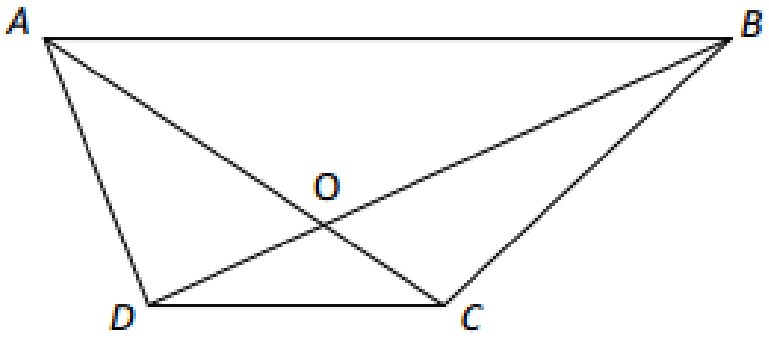

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Triangle-based Explorations

Testing Conjectures

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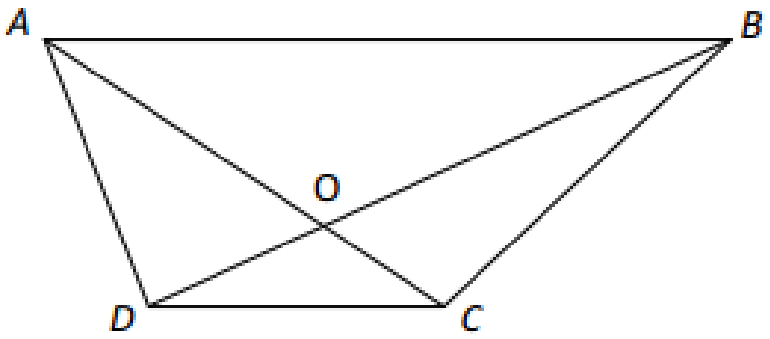

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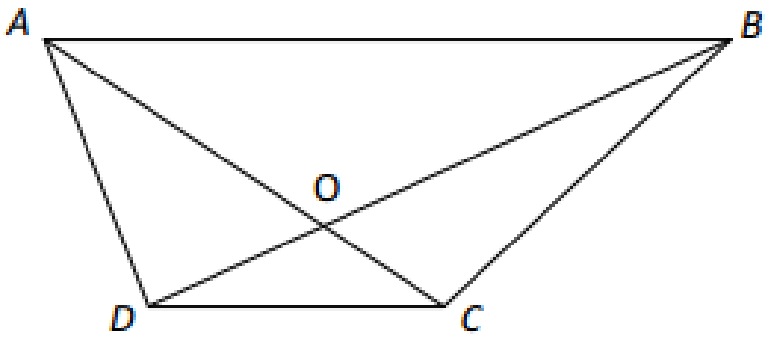

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Triangle-based Explorations

Manipulating Area

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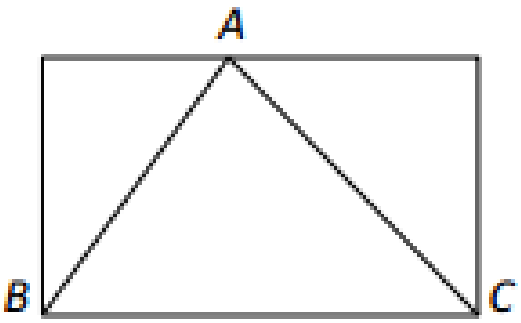

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