Module 5

**Topic: Triangle-based Explorations**

The purpose of this professional development session is to provide teachers the opportunity to explore a typical calculus problem using geometric approaches in an effort to expand their current problem-solving task inventories to include higher level mathematics problems. Teachers will also practice creating and testing conjectures, and consider students work samples to the same problems.

**Mathematical Elements**

- Consider the relationships between and among triangles and polygons, and associated properties.
- Investigating different approaches (i.e. geometric) to solve probability problems.
- Probability can be interpreted geometrically as proportional to the area defined by constraints.
- Practice creating and testing conjectures
- Triangle area, congruence, similarity, proportion, random selection

**Pedagogical Elements**

- Classification of problems (i.e. the spaghetti problem is a calculus problem) is not a productive way of thinking about instruction and limits the problems we offer to students.
- Problem-solving tasks can become accessible to all children if different approaches and techniques are used.
- Technology affords the space for conjecturing and testing so students can draw their own inferences.

**Activities**

**Task**

- A stick of uncooked spaghetti is broken into three pieces by picking two points independently and uniformly along the stick, and breaking the stick at those two points. What is the probability that the three pieces can be assembled to form a triangle?
- Post solution strategies on chart paper.

**Task**

- The area of the rectangle is 1 square inch. Describe how the area of triangle $ABC$ would change when $A$ moves along the side of the rectangle.

- Discuss student work samples.
Task

- \(ABCD\) is a trapezoid with \(AB\) parallel to \(DC\). The diagonals \(AC\) and \(BD\) intersect at \(O\). Make a few conjectures about the four triangles formed inside trapezoid \(ABCD\). Explain how you arrived at these conjectures.

![Diagram of trapezoid ABCD with diagonals AC and BD intersecting at O]

- Discuss student work samples.

Task

- Dinglei claims that \(\triangle COB\) and \(\triangle ADO\) have the same area and that \(\triangle DOC\) and \(\triangle AOB\) are similar. However, she does not provide an explanation for her claims. Offer an explanation for why you believe she is right or wrong in each case.

![Diagram of trapezoid ABCD with diagonals AC and BD intersecting at O]

- Discuss student work samples.

Task

- The area of the rectangle is 1 square inch. Assuming \(\triangle ABC\) shares the side \(BC\) with the rectangle, and vertex \(A\) of the triangle remains on the opposite side of \(BC\), is it possible for the area of \(\triangle ABC\) to be larger than one half of the area of the rectangle?

![Diagram of a rectangle with \(\triangle ABC\) inside]

- Discuss student work samples.

Guided Questions for Debrief

- What other higher-level mathematics problems can be solved using different approaches that we would add to our inventory?
- What questions could teachers ask students that extend the concept horizontally?
- What questions could teachers ask students that extend the concept vertically?
Which Mathematical Practices were demonstrated with these tasks?

**Materials**

**Technology**
- Laptop
- Document Projector
- Document Camera

**Supplies**
- Chart Paper/Markers
- Copies of handouts for each separate task
- Copies of Student Work Samples handout
- Raw spaghetti noodles
Triangle-based Explorations

Spaghetti Problem
A stick of uncooked spaghetti is broken into three pieces by picking two points independently and uniformly along the stick, and breaking the stick at those two points. What is the probability that the three pieces can be assembled to form a triangle?
Triangle-based Explorations

Changing Area
The area of the rectangle is 1 square inch. Describe how the area of triangle $ABC$ would change when $A$ moves along the side of the rectangle.
Student Work Samples

Changing Area
The area of the rectangle is 1 square inch. Describe how the area of triangle $ABC$ would change when $A$ moves along the side of the rectangle.
Triangle-based Explorations

Conjectures about Triangles

$ABCD$ is a trapezoid with $AB$ parallel to $DC$. The diagonals $AC$ and $BD$ intersect at $O$. Make a few conjectures about the four triangles formed inside trapezoid $ABCD$. Explain how you arrived at these conjectures.
Student Work Samples

Conjectures about Triangles

$ABCD$ is a trapezoid with $AB$ parallel to $DC$. The diagonals $AC$ and $BD$ intersect at $O$. Make a few conjectures about the four triangles formed inside trapezoid $ABCD$. Explain how you arrived at these conjectures.
Triangle-based Explorations

Testing Conjectures
Dinglei claims that ΔCOB and ΔADO have the same area and that ΔDOC and ΔAOB are similar. However, she does not provide an explanation for her claims. Offer an explanation for why you believe she is right or wrong in each case.
Student Work Samples

Testing Conjectures
Dinglei claims that $\triangle COB$ and $\triangle ADO$ have the same area and that $\triangle DOC$ and $\triangle AOB$ are similar. However, she does not provide an explanation for her claims. Offer an explanation for why you believe she is right or wrong in each case.
**Triangle-based Explorations**

**Manipulating Area**

The area of the rectangle is 1 square inch. Assuming $\triangle ABC$ shares the side $BC$ with the rectangle, and vertex $A$ of the triangle remains on the opposite side of $BC$, is it possible for the area of $\triangle ABC$ to be larger than one half of the area of the rectangle?
Student Work Samples

Manipulating Area
The area of the rectangle is 1 square inch. Assuming $\triangle ABC$ shares the side $BC$ with the rectangle, and vertex $A$ of the triangle remains on the opposite side of $BC$, is it possible for the area of $\triangle ABC$ to be larger than one half of the area of the rectangle?