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Module 5

Topic: Authentic Mathematical Modeling

The purpose of this session is to provide 9-12th grade teachers with a few opportunities to engage in the process of mathematical modeling to solve authentic problems rooted in the real world. The primary focus is for teachers to develop and refine mathematical models as student learners to provide them a sense of what it might look like when implemented in their classroom.

Mathematical Elements

- Applying mathematics to solve mathematical modeling problems
- Generating mathematical representations of a real-world system (mathematizing space)
- Learning about assumption-making and controlling for variables in a modeling problem

Pedagogical Elements

- Developing skills in facilitating mathematical modeling activities
- Helping students in making assumptions to control for the large number of variables present in authentic modeling problems

Activities

Tasks and Guided Questions for Discussion

- **Discussion: Definition of Mathematical Modeling –** This activity provides an opportunity for teachers to discuss their ideas about what mathematical modeling is, and why it is important. The purpose of this session is to provide the teachers with a common definition for mathematical modeling and exposure to authentic mathematical modeling tasks. This discussion builds upon the previous sessions as problems encountered have become more open to multiple solution strategies and a variety of answers. This space will be used for teachers to ask questions about mathematical modeling and frame the Kroger Fuel Point Problem activity.

Mathematical Modeling Contexts – The purpose of this task is to expose the classroom teachers to a variety of modeling contexts to explore. Teachers will be split into groups of 3-4, and provided a minimum of two modeling contexts (listed below). Following the completion of these two tasks, responses will be shared to the entire groups. The purpose of this activity is to generate a discussion about the skills needed in solving these problems, and how these may be enacted in a classroom environment. The teachers will be asked to provide a mathematical and pedagogical analysis of each problem context. Support staff will be available in the solving of these problems to help push teachers mathematically and assist them in considering and accounting for alternate variables.

- **Kroger Fuel Points Problem –** Give the teachers a copy of the *Kroger Fuel Points Handout* and put them into groups of four. Allow the teachers to work through thinking about how

to maximize their savings for the year using the constraints set. Once groups have constructed a model of the problem scenario, teachers should be encouraged to engage in additional iterations of the mathematical modeling cycle to refine their model. OSU Mathematician (Fowler) will be available to help groups extend their mathematical representations of the problem scenario. Two things to highlight: what assumptions were made in constructing the model and what variables did the model account for. The teachers should be asked at the end:

- What factors did you originally consider when coming up with a solution? Are there any that you dismissed, and why?
- Would you implement your strategy? Why or why not?
- Is there anything that could improve your strategy?

- **Rainfall Problem –** Give teachers a copy of the *Rainfall Problem Handout*. In this context teachers are asked to explore whether it would be more beneficial to walk or run if caught in a rainstorm. What factors can determine this? Are there better times to walk versus run? Build a model of the problems scenario and use it to determine for what conditions would be more optimal to walk versus run.
- **Cost of Me (Dossey) –** Build a model to determine the "cost of me." This problem could be interpreted from a variety of standpoints (elements that make a person up, cost of your economic footprint, budgeting, determination of how much money is spent by others to sustain you, etc.). Construct a mathematical model to help you determine the cost. In sharing out to the large group, teachers should emphasize how they interpreted the question, and how they went about solving it. As they work, teachers should be mindful of the mathematics required to determine a solution to this context as well as how this type of problem might be implemented in a classroom.
- **Postman's Letter Problem -** Suppose that a street has 10 mailboxes on each side of the street. A postman in delivering letters can either walk along one side of the street until they reach the end and cross walking back the other way, or can deliver to a mailbox, cross the street between mailboxes. Can you determine an optimal path for the postman to take to deliver his mail? What assumptions did you account for in your solution?
- **Concert Problem -** A rock band is putting on a concert at your local theater, and you have been hired to determine how much to charge for tickets. You have been provided a layout of the floor plan of the venue. What factors will you account for in order to determine a ticket price? What additional information do you need in solving this problem? The last venue they played at offered you a photo of the crowd to assist you in your planning. How can this be used to determine the price of each ticket?
- **Pretzel Jar Problem** – Teachers will remain in their work groups from previous activity and given a copy of the *Pretzel Jar Problem Handout*. In this activity teachers will be asked to describe at least three methods to determine the number of pretzels in a jar. Teachers will then rank their approaches by level of mathematical sophistication, and consider how their 9-12 students might accomplish this task. Again in implementation of this task teachers will be asked to consider those assumptions made during the activity and identify which variables they decided to control for in each iteration of the task.
- **Pond Design Problem – [get info from Monelle or Dr. M]**

Modeling Gallery Walk: After teachers have had an opportunity to work on two modeling contexts, teachers will share their solutions via gallery walk by hanging their work on the walls. One or two teachers from each group will stay with their work while others visit the work of other groups to get a sense for how different groups tackled the modeling contexts. After 10 minutes of visiting other groups, the pairs of teachers will switch so that all teachers will have an opportunity to see and describe their work.

Following the gallery walk, a brief discussion will be had surrounding similarities and differences in the solutions to the problem. This will lead into the guided debrief of the modeling scenario (questions listed below).

Guided Questions for Debrief

- In what ways did you identify variables that would impact your model?
- How did you identify and select (as a group) which variables were the most important to account for in your model?
- What types of knowledge about the system being modeled did you draw on in this activity?
- What mathematical concepts did you use when constructing your model?
- How might your students react to these problems? What variables do you anticipate would they account for in their model?
- How would you facilitate this type of task in your classrooms?
- When in instruction do you see this activity being the most effective?
- What barriers might you confront in implementing this task in a classroom?
- When teaching mathematical modeling, how can we prepare to assist students in this process?
- Was your model similar to other groups or drastically different? Is this type of discrepancy productive? Why?
- How did this mathematical modeling task differ from traditional application problems that have been used previously?

Materials

Technology

- Laptop
- iPads
- Document Projector
- Document Camera

Supplies

- Chart Paper
- Markers
- Activity handouts

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Kroger Fuel Points

When you shop at Kroger with a Kroger card, you can earn 1 fuel point for every $1 you spend. For each 100 points you save 10 cents per gallon of fuel, up to $1 per gallon. Each month is a separate accumulation period and points do not combine. The points expire at the end of the following month (i.e. the fuel points from what you spent in March will expire at the end of April).

When you go to a Kroger Fuel Center with a Kroger card, you automatically save 3 cents per gallon of fuel. In addition, if you have 100 or more fuel points accumulated, you can choose how much of your discount you want to use in increments of 10 cents, so 10¢, 20¢, 30¢, etc.

For example, if you spent a total $320 in the month of March then you have 30 cents off as your discount. You could take all 30 cents off of one tank of gas now. You could leave it for another time (and so only get the usual 3 cent discount this time). You could take 10 cents off now, 10 cents off another time, and 10 cents off another time to spread it out. You could even take 10 cents off now and 20 cents off later. You get to choose.

Develop a strategy to maximize your savings on gas from Kroger Fuel Points over the course of a year and explain why it will maximize your savings.

Follow up Questions:

1) What factors did you originally consider when coming up with a solution? Are there any that you dismissed and why?

2) Would you implement your strategy? Why or why not?

3) Is there anything that could improve your strategy?

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Rainfall Problem

Suppose you are out walking without an umbrella and it starts to rain. If you want to minimize how wet you get during your walk, would it be more beneficial to walk or run and under what conditions?

Develop a strategy to minimize how wet you get during your travel.

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The Cost of Me

Your task is to develop a mathematical model that can determine the "the cost of me". This could be interpreted in a number of ways.

1. Think of at least three different ways that you could interpret what this problem is asking. Of these interpretations, select one of these that you wish to explore.
2. Based on your interpretation of the problem, think about what variables you need to consider in developing your model? What assumptions do you have to make regarding this situation?

Develop a model that you could use to determine *The Cost of Me* based on your interpretation of the problem.

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The Postman's Problem

On is delivery route a postman has to deliver to mail boxes on both sides of the street. The postman can either deliver all the letters to one side of the street, cross, then deliver to the other side of the street, OR deliver to one mailbox cross the street and deliver to another one or two mailboxes, then cross the street again. Under what conditions are either of these the optimal methods for delivery? What assumptions did you make in accounting for this problem?

Develop a model to determine the optimal route for the postman to travel on his mail delivery route.

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The Concert Problem

A rock band is putting on a concert at your local theater, and you have been hired to determine how much to charge for tickets. You have been provided a layout of the floor plan of the venue. What factors will you account for in order to determine a ticket price? What additional information do you need in solving this problem? The last venue they played at offered you a photo of the crowd to assist you in your planning. How can this be used to determine the price of each ticket?

Develop a strategy for determining how much to charge for each ticket and justify your decision.

Nationwide Arena, Columbus Ohio

2015 Mathematics Teaching Institute

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Pond Design Problem

There is a new residential development that is being constructed, and the designers want to include a pond. Suppose that the residential development will only include 5 story apartment complexes, and that the community will include approximately 300 families (at maximum capacity). How large should the pond be and what will be the design? The total square footage of the complex is 360,000 square feet.

Develop a strategy to determine how large the pond should be in the apartment complex. What factors should you consider in determining a solution to this task? Justify your solution. What other questions arise in solving this problem?

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Pretzel Jar Task

You brought a big jar of pretzels to share at a party. They are a huge hit and everyone is snacking on them! Whenever you walk by, you notice less and less pretzels in the jar. How would you estimate the number of pretzels left in the jar at any time during the party? (Don't forget about the broken pieces!)

Task:

- Describe **three** methods to estimate the number of pretzels in the jar.
- Rank your three approaches by the level of mathematical sophistication.
- Describe how you anticipate students in different grade bands would estimate the number of pretzels in the jar.

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Investment Task

Why should I invest? "*One of the most compelling reasons for you to invest is the prospect of not having to work your entire life! Bottom line, there are only two ways to make money: by working and/or by having your money work for you*" – Investopedia.

Money can earn you money in two ways. First, you give someone money for a period of time and that person pays you back with interest. Second, when you give someone money, in return you are a part owner of their business. In other words, you now possess some financial assets. This is investing!

Task:

You have $1,000 and two friends with business plans. Suppose you like them equally as friends. Both business plans sound exciting, but ultimately your goal is to make money off of your investment. If you have to choose only one, how do you pick the business plan to invest in?

Write an investment plan that includes an outline of what you consider necessary data for your choice of investment (i.e. factors that would affect investment choices).

- Select the most viable factors and explain why each piece of information is important to the comparisons.
- Rank the importance of each piece of information for your investment decision because you may not be able to collect all information that you need.
- Build mathematical connections of each factor for investment decision making.
- Use spreadsheet to build an investment calculator to help you decide in which business to invest.

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College Saving Problem

Suppose you want to start a college savings plan for your child(ren). Consider a variety of options and determine an optimal method for savings. How much money will you be putting away per paycheck? How much money will you save by the time your children are in school? How can you maximize the amount of money to put away?

Develop a strategy for savings, and determine how much money you will have saved by the time your children finish school.