Module 2

Topic: Mathematical Representations

The purpose of this professional development session is to solve mathematical tasks that have potential to be solved using multiple representations. Teachers will practice abandoning typical, but unproductive, solution paths in an effort to increase flexibility between mathematical domains. Teachers will become familiar with the distinguishing characteristics of ratios and proportions and the connections of these mathematical concepts across the curriculum.

Mathematical Elements
- Consider the distinguishing characteristics of ratios and proportions.
- When we talk about graphing ratios, would the graph be continuous or discrete, and why?
- Is graphing a relationship defined by a ratio different from graphing a proportional relationship?
- Argumentation and reasoning with mathematical models and representations.
- Increase flexibility in representation use when one method is more productive than another.

Pedagogical Elements
- Mathematical practice
- Honoring student thinking and multiple representations.
- Student tend to betray their own thought process by dismissing own idea by expectations of the environment. They dismiss their own intuition that is right on target, but it might not fit the model the teacher uses. What are the key important things that teachers need to become sensitive to in order to gradually develop in student thinking?
- If instruction emphasizes only one model or one representational type, then the mathematical tools that children acquire is limited.

Activities
Task

- The ratio of boys to girls in a class is 5 to 3. After six girls join the class, the number of boys and girls in the class is the same. How many students are in the class now?
- Post solution strategies on chart paper.

Student Work Samples for Ratio of Boys to Girls Task

- Is child cognizant that there are more answers or does he just stop because he thinks the problem is solved? Does he understand the cardinality of the solution set?
- What do we learn about each child based on their responses? Focus not on what the child does not know but what the child seems to know.
- What do we learn from the children's work? Which of the answers were similar to yours?

Task

- The ratio of girls to boys in a freshman class is 4:3. The ratio of girls to boys in a sophomore class is 5:4. If we combine the two classes, what would be the ratio of girls to boys in the group?

Task

- Terry has an appointment with his barber. He decides to ride his bicycle to the barber shop instead of driving. If he rides his bike at 10 miles per hour, then he gets there an hour late for his appointment. However, if he rides his bike at 15 miles an hour, then he gets there an hour early. How far does Terry live from the barber shop?

Task

- A pie graph has four unequal slices representing favorite soft drink preferences for a sample of n people. Eight more people join the sample and three-fourths of them choose Coca-Cola as their favorite soft drink. This result makes the Coca-Cola slice increase to exactly 54% of the total graph. How many people were initially in the sample?

Task

- A survey about favorite desserts reports:
  12% of the sample prefer cake
   4% of the sample prefer cookies
  28% of the sample prefer ice cream
  40% of the sample prefer chocolate
  What is the smallest number of people that could have taken the survey?

Task

- If $\frac{b}{a} = 2$ and $\frac{c}{b} = 3$, what is the ratio of $a + b$ to $b + c$? 60% of the members of a computer club were girls. Later, 15 boys joined the club. With the addition of boys, do you expect the percentage of girls to be larger than 60% or smaller? Please explain your reasoning.

Task

- 60% of the members of a computer club were girls. Later, 15 boys joined the club. With the addition of boys, do you expect the percentage of girls to be larger than 60% or smaller? Please explain your reasoning.
- Is it possible to find the percentage of female members in the computer club after the addition of boys? If so, what is the new percentage of females? If not, explain what other information you would need to determine the percentage of girls in the computer club.

Task

- As the star player of a basketball game stands at the free throw line, the announcer states that he had hit 78% of his free throws that year. The star player misses the first shot and makes the second. Later in the game he is fouled for the second time. As he moves to the free throw line, the announcer states that he had made 76% of his free throws so far that year. Can you determine how many free throws this player had attempted and how many he had made that year?

Guided Questions for Debrief

- How might we make the different types of representations accessible to students?
- What other problems might be used to help students acquire these schemes?
- What questions could teachers ask students that extend the concept horizontally?
- What questions could teachers ask students that extend the concept vertically?
- Which Mathematical Practices were demonstrated with these tasks?

Materials
Technology

- Laptop
- Document Projector
- Document Camera

Supplies

- Chart Paper/Markers
- Copies of handouts for each separate task
- Copies of Student Work Samples handout

Modeling with Mathematics

Ratio of Boys to Girls

The ratio of boys to girls in a class is 5 to 3. After six girls join the class, the number of boys and girls in the class is the same. How many students are in the class now?

Modeling with Mathematics

Ratio of Girls to Boys

The ratio of girls to boys in a freshman class is 4:3. The ratio of girls to boys in a sophomore class is 5:4. If we combine the two classes, what would be the ratio of girls to boys in the group?

Modeling with Mathematics

Terry's Trip to the Barber

Terry has an appointment with his barber. He decides to ride his bicycle to the barber shop instead of driving. If he rides his bike at 10 miles per hour, then he gets there an hour late for his appointment. However, if he rides his bike at 15 miles an hour, then he gets there an hour early. How far does Terry live from the barber shop?

Modeling with Mathematics

Favorite Soft Drink Pie Graph

A pie graph has four unequal slices representing favorite soft drink preferences for a sample of *n* people. Eight more people join the sample and three-fourths of them choose Coca-Cola as their favorite soft drink. This result makes the Coca-Cola slice increase to exactly 54% of the total graph. How many people were initially in the sample?

Mathematical Modeling

Favorite Desserts

A survey about favorite desserts reports:
- 12% of the sample prefer cake
- 4% of the sample prefer cookies
- 28% of the sample prefer ice cream
- 40% of the sample prefer chocolate

What is the smallest number of people that could have taken the survey?

Modeling with Mathematics

Ratio Challenge

If $\dfrac{b}{a} = 2$ and $\dfrac{c}{b} = 3$, what is the ratio of $a + b$ to $b + c$?

Modeling with Mathematics

Computer Club

60% of the members of a computer club were girls. Later, 15 boys joined the club. With the addition of boys, do you expect the percentage of girls to be larger than 60% or smaller?

Is it possible to find the percentage of female members in the computer club after the addition of boys? If so, what is the new percentage of females? If not, explain what other information you would need to determine the percentage of girls in the computer club.

Modeling with Mathematics

Basketball Player

As the star player of a basketball game stands at the free throw line, the announcer states that he had hit 78% of his free throws that year. The star player misses the first shot and makes the second. Later in the game he is fouled for the second time. As he moves to the free throw line, the announcer states that he had made 76% of his free throws so far that year. Can you determine how many free throws this player had attempted and how many he had made that year?

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Student Work Samples

The ratio of boys to girls in a class is 5 to 3. After six girls join the class, the number of boys and girls in the class is the same. How many students are in the class now?

Student #1

I took $\frac{5}{8}$ and multiplied it by 2 which gives me $\frac{10}{16}$ then I added 6 (the number of new girls) to it and it gives me $\frac{16}{16}$, the number of students in the class is 32, 16 girls & 16 boys.

Student #2

# in class	B	G
8	5	3
16	10	6
24	15	9
32	20	12
40	25	15
48	30	18

# of girls plus 6 which will equal the # of boys

$5 \neq 3+6$
$10 \neq 6+6$
$\boxed{15 = 9+6}$

Student #3

$\frac{5}{8}$ = boys + 6 gil

$\frac{5}{8} + \frac{3}{8} = \frac{8}{8}$

so

$\frac{5}{8} = \frac{3}{8} + 6$

$\frac{5}{8} + 6 = A$

$\frac{5}{8} =$

I don't know

Student #4

Student #5

I looked at where they were the same (b=g). That was when we had 15 of each. So at the beginning there must have been 24 ~~stu~~ students
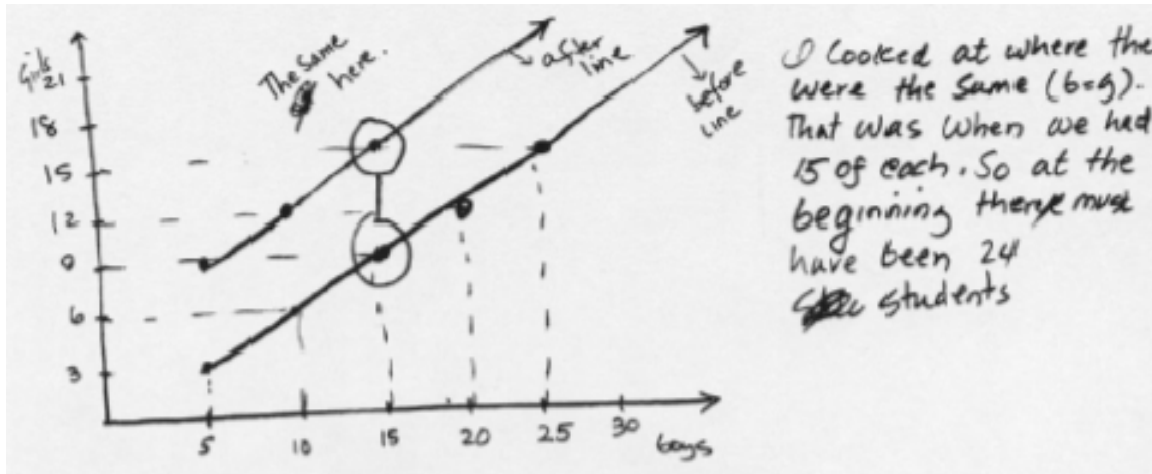
Student #6

The third one works because when we give two girls to each pair of ~~girls~~ girls then rows of boys and girls are ~~equal~~ equal.