The Ohio State University

Problem Posing

Module 4

Mathematics Teaching Institute, July 27-31, 2015

Sponsored by Ohio Department of Education

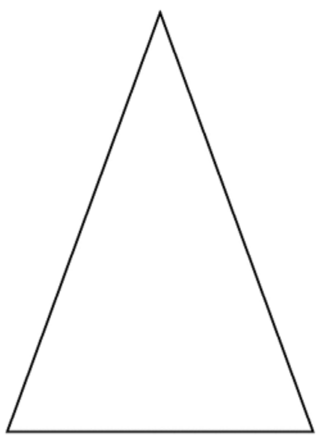
Brown & Walter (2005)

Two phases of problem posing:

- Accepting the given problems
- What if not…?

(Brown & Walter, 2005)

An Isosceles Triangle

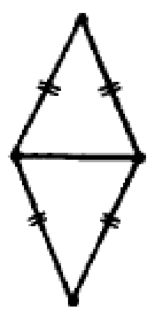


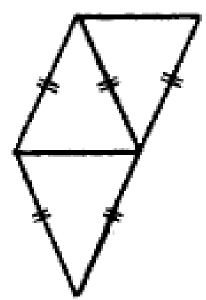
Assume that we have a triangle in which two sides are equal in length— an isosceles triangle. What questions could you ask?

1. How might we classify isosceles triangles? We might, for example, classify them with regard to their vertex angles (e.g., obtuse, right, acute) or the ratio of the lengths of base to side. What other ways can you think of?
2. What types of symmetry does an isosceles triangle have?
3. If one angle of an isosceles triangle is twice another, is the shape of the triangle determined?
4. What relationships exist among the exterior angles of the triangle? How do the exterior angles relate to the interior angles?

5. What was it that encouraged people initially to investigate isosceles triangles?

6. What figures can you make with congruent isosceles triangles? Using two of them? Using three of them? Others? What geometric figures have been created below by replicating an isosceles triangle? Can you make others?





(Brown & Walter, 2005)

Generate conjectures, questions and observations:

9, 16, 23, 30, 37, 44, 51, 58, ...

- Two numbers of the list given are prime. As you extend the sequence will there be an infinite number of primes?
- The numbers alternate between ones that are odd and even.
- There is a number in the sequence that is divisible by 2, a number divisible by 3, one by 4, one by 5, by 6, but not one by 7. Is 7 the only exception?
- Do all digits from 0 to 9 occur in the units place? Tens place?
- Is there a pattern to the last digits?
- Can you tell quickly if 1938 appears in the list?

(Brown & Walter, 2005)

A Handy List of Questions

- Is there a formula?
- What is the formula?
- What purpose does the formula serve?
- What is the number of objects or cases satisfying this condition?
- What is the maximum?
- What is the minimum?
- What is the range of the answer?

- Is there a pattern here?
- What is the pattern in this case?
- Is there a counterexample?
- Can it be extended?
- Does it exist?
- Is there a solution?
- Can we find the solution?
- How can we condense the information?
- Can we make a table?

- How can you view it geometrically?
- How can you view it algebraically?
- How can you view it analytically?
- What do they have in common?
- What do I need in order to prove this?
- What are key features of the situation?
- What are the key constraints currently being imposed on the situation?
- Does viewing actual data suggest anything interesting?
- How does this relate to other things?

Brown & Walter (2005)

Two phases of problem posing:

- Accepting the given problems
- What if not…?

Adapting Task

Find the area of a rectangle whose dimensions are 3 feet by 4 feet.

- Solve it!
- What was given?
- What were you being asked to find?

Your garden has an area of 20 square feet. What are possible dimensions of the garden?

- Solve it!
- What was given?
- What were you being asked to find?

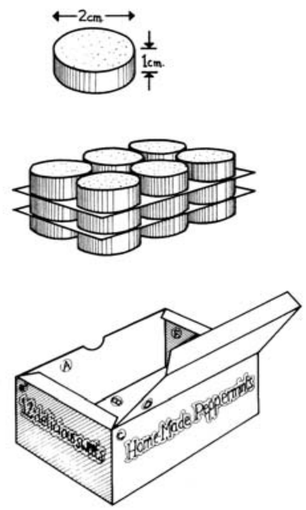
Sally has 128 feet of fence and wants to enclose an area in her backyard so her dog can run around. How should she position the fence so the dog will have the most area to run and play?

- Solve it!
- What was given?
- What were you being asked to find?

What do you notice in the progression of the three tasks?

Problem Posing

Designing a box for 18 mints



- Work through the structured problems carefully.
- List all the decisions that are being made.
- Revise the problem so that some of these decisions are handed back to students. This will make them less structured.

Handout 2 contains unstructured versions of the same task.

- Compare the less structured versions of the problems with the structured versions.
- What decisions have been left to the students?
- What pedagogical issues will arise when you start to use unstructured problems like this?

What is role of the teacher in problem solving lessons?

Handout 3 contains some practical advice when using unstructured problems. Consider this advice carefully:

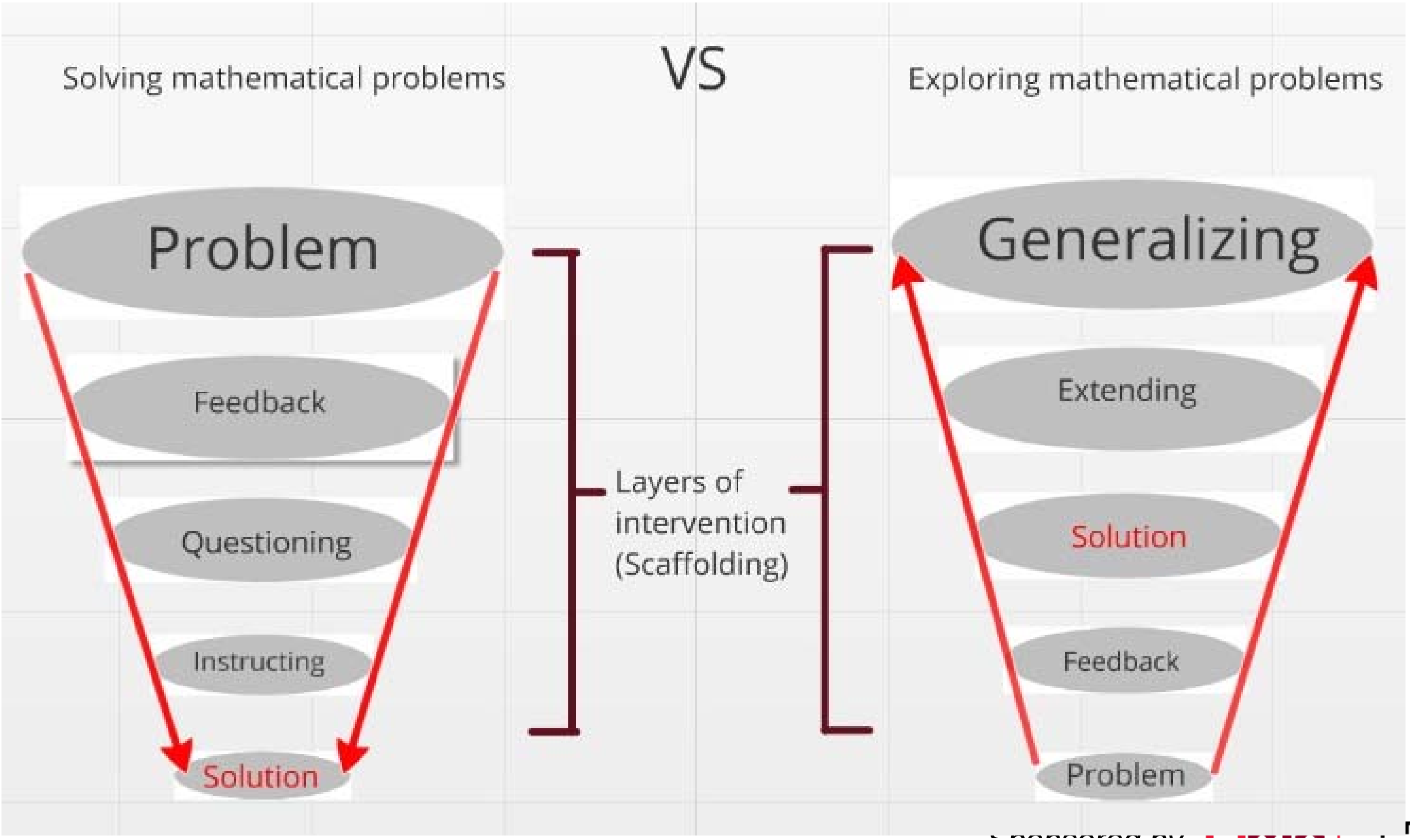
- Which ideas do you normally find most difficult to implement? Why is this?
- Is there any other advice you would add to this list? Write your own ideas at the bottom.

Scaffolding

"Any failure by a child to succeed in an action after a given level of help should be met by an immediate increase in help or control. Success by a child then indicates that any subsequent instruction should offer less help than that which preceded the success, to allow the child to develop independence." (Wood, 1998)

Six Levels of Scaffolding Wood, Bruner, and Ross (1976)

1. *Recruitment:* the adult encourages the child to engage in the task at hand, and spark the child's interest in the task.
2. *Reduction:* the adult simplifies the task in order to meet the child's current cognitive level
3. *Direction maintenance*: the adult verbally encourages the child to seek a solution to the task or an understanding of the learning objective.
4. *Marking critical features:* the adult interprets the child's work or solution and intervenes to dispel misconceptions or misunderstandings.
5. *Frustration control:* If the child shuts down or becomes frustrated with the learning process, the adult mediates the child's obstruction in order to achieve the desired goal.
6. *Demonstration:* The adult models the same or similar task in order to assist the child's development and understanding.


Solving mathematical problems
VS
Exploring mathematical problems
Problem
Feedback
Questioning
Instructing
Solution
Layers of intervention (Scaffolding)
Generalizing
Extending
Solution
Feedback
Problem

Plan a Lesson, Teach it and Reflect on the Outcomes

Choose a problem that you bring here or any problems that you solved here. A problem that you feel would be appropriate for your class.

Discuss how you will:

- Make it less structured, if necessary.
- Organize the classroom and the resources needed.
- Introduce the problem to students.
- Explain to students how you want them to work together.
- Challenge/assist students that find the problem straightforward/difficult.
- Help them share and learn from alternative problem-solving strategies.